

RANDOM SHOOTING OF ENTANGLED PARTICLES IN VACUUM

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The effect of random shooting of particles is considered on the basis of solution of the Schrödinger equation and in terms of the Wigner function. Two-particle description shows, in particular, that initial correlation leads to high velocities of particles. This could be a potential mechanism for obtaining energy. Evolution of the n -particle probability distribution is described analytically.

The Heisenberg's uncertainty relation reads:

$$\Delta p \Delta x \sim \hbar \quad (1)$$

where Δp is uncertainty of momentum ($p = mv$ - product of mass and velocity of particle), Δx is uncertainty of coordinate and \hbar is Planck's constant. Relation (1) seems to preclude the existence of trajectory: the more exactly we know location the more uncertain is velocity. Probability density of particle position is given by $P = \Psi\Psi^*$, where $\Psi(t, x)$ is the wave function and star indicates complex conjugate. For simplicity, we start with the one-dimensional case, but the 3-D generalization is simple (see below).

Schrödinger's equation has the form (see, for example, Ref. 1):

$$i\hbar\partial\Psi/\partial t = -(\hbar^2/2m)\partial^2\Psi/\partial x^2 + U(x)\Psi \quad (2)$$

Here $U(x)$ is the potential energy of the particle in the external field, i is the imaginary unit. The strange behavior of particles is not associated with particular form of potential energy. So, we put $U = 0$ and consider the influence of the quantum vacuum on the motion of particle. Equation (2) becomes very simple:

$$\partial\Psi/\partial t = iq\partial^2\Psi/\partial x^2, \quad q \equiv \hbar/2m \quad (3)$$

where q is the coefficient of imaginary diffusion (ID). Equation for Ψ^* has the same form as (3) with minus in the right hand side.

General solution of equation (3) in unbounded space can be obtain with use of Fourier transform:

$$\partial\tilde{\Psi}/\partial t = -iqk^2\tilde{\Psi} \quad (4)$$

where k is the wave number and $\tilde{\Psi}(t, k)$ is the transform of $\Psi(t, x)$. Solution of (4) is:

$$\tilde{\Psi}(t, k) = \tilde{\Psi}_0(k) \exp\{-iqk^2t\} \quad (5)$$

where zero indicates initial function. The inverse transform of (5) gives:

$$\Psi(t, x) = (4\pi iqt)^{-1/2} \int_{-\infty}^{\infty} dx' \Psi_0(x') \exp\left\{-\frac{(x-x')^2}{4iqt}\right\} \quad (6)$$

We would like to clarify the physical sense of ID and expressions (5) and (6).

Let us introduce Gaussian random velocity $v(t)$ with zero mean ($\langle v(t) \rangle = 0$) and correlation proportional to δ -function (see, for example, textbook [2]):

$$\langle v(t+\tau)v(t) \rangle = q\delta(\tau) \quad (7)$$

Here and below brackets $\langle \rangle$ indicate statistical averaging. Consider trajectory, produced by such velocity:

$$z(t) = z_0 + \int_0^t v(t') dt' \quad (8)$$

We assume that initial position z_0 has Gaussian distribution, independent of $v(t)$, with zero mean ($\langle z_0 \rangle = 0$) and dispersion $\langle (z_0)^2 \rangle = a^2$. Then, $z(t)$ will have Gaussian distribution with zero mean and dispersion:

$$\langle [z(t)]^2 \rangle = a^2 + qt \quad (9)$$

Here we used (7) and independence of $v(t)$ from z_0 .
Now, consider complex trajectory:

$$y(t) = y_0 + (1+i) \int_0^t v(t') dt' \quad (10)$$

Here y_0 is real with the same statistics as z_0 . We have $\langle y(t) \rangle = 0$ and

$$\langle [y(t)]^2 \rangle = a^2 + 2iqt \quad (11)$$

High order statistical moments of $y(t)$ can be calculated from these first two by the same formulas as for the Gaussian distribution. Effective probability density (epd) for $y_1 = y - \langle y \rangle$

$$(2\pi \langle y_1^2 \rangle)^{-1/2} \exp\left\{-\frac{y_1^2}{2 \langle y_1^2 \rangle}\right\}$$

can be used, as if y is real. For statistical moments of $y^*(t)$ in all formulas we have to change the sign in front of i . We will call such complex random processes complex-Gaussian.

Returning to (6), for simplicity of calculations, let us choose:

$$\Psi_0(x) = (2\pi a^2)^{-1/4} \exp\{-x^2/4a^2\} \quad (12)$$

This wave function corresponds to the Gaussian probability density for the initial position of particle:

$$P_0(x) = (2\pi a^2)^{-1/2} \exp\{-x^2/2a^2\} \quad (13)$$

Substitution of (12) into (6), after simple calculation, gives:

$$\Psi(t, x) = \frac{a^{1/2}}{(2\pi)^{1/4}(a^2 + iqt)^{1/2}} \exp\left\{-\frac{x^2}{4(a^2 + iqt)}\right\} \quad (14)$$

This expression (apart from normalizing factor) is the epd for the complex-Gaussian process like (10) with a replaced by $\sqrt{2}a$. It suggests that quantum particles have hidden complex trajectories. It also suggests that the quantum vacuum interacts with particles by producing tachyonic impulses (with imaginary energy), which leads to ID (see below).

Using (14), we calculate the probability density for the position of particle in real world:

$$P(t, x) = \Psi\Psi^* = \frac{1}{[2\pi(a^2 + w^2t^2)]^{1/2}} \exp\left\{-\frac{x^2}{2(a^2 + w^2t^2)}\right\} \quad (15)$$

Here $w = qa^{-1}$ is effective constant velocity. The trajectory for (15) is random shooting:

$$x(t) = x_0 + ut \quad (16)$$

where x_0 and u are independent random constants with Gaussian distributions and $\langle x_0 \rangle = 0$, $\langle (x_0)^2 \rangle = a^2$, $\langle u \rangle = 0$, $\langle u^2 \rangle = w^2$. The fact that w is inversely proportional to a is, of course, manifestation of the uncertainty relation (1).

Note that if we multiply the initial wave function (12) by $\exp\{i\gamma x\}$, where γ is real constant, the initial probability density (13) will not change. However in (15) x will be replaced by $x - 2q\gamma t$, so we will have $\langle x \rangle = 2q\gamma t$. In this paper we choose initial conditions such that the mean positions of particles will not change in time.

The quadratic time-dependence of dispersion in (15) shows that quantum complex trajectories like (10) can not be just projected on real axis, leading to linear time-dependence (9). The nonlinear operation $\Psi\Psi^*$ in (15) is proportional to the joint epd for the complex and complex conjugate trajectories, as if they are independent. This indicates an interesting sort of interaction. It is remarkable that such interaction produces random shooting (16). Note that the effective energy $mw^2/2$ is inversely proportional to m .

Let us look into details of this interaction by considering the characteristic function:

$$\Phi(t, k) = \int dx P(t, x) \exp\{ikx\} = \int dx \Psi(t, x) \Psi^*(t, x) \exp\{ikx\} \quad (17)$$

Using Fourier transformation, we get from (17):

$$\Phi(t, k) = \frac{1}{2\pi} \int dm \tilde{\Psi}(t, m) \tilde{\Psi}^*(t, m - k) \quad (18)$$

The transform of (12) is:

$$\tilde{\Psi}_0(k) = 2^{3/4} \pi^{1/4} a^{1/2} \exp\{-a^2 k^2\} \quad (19)$$

Solution (5) gives:

$$\tilde{\Psi}(t, k) = 2^{3/4} \pi^{1/4} a^{1/2} \exp\{-(a^2 + iqt)k^2\} \quad (20)$$

From (18) we now have:

$$\Phi(t, k) = 2^{1/2} \pi^{-1/2} a \int dm \exp\{-a^2[m^2 + (m - k)^2] - iqt[m^2 - (m - k)^2]\} \quad (21)$$

For $t = 0$:

$$\Phi(0, k) = \exp\{-\frac{1}{2}a^2 k^2\} \quad (22)$$

which corresponds to the transform of (13).

Let us stress the difference between the ordinary diffusion, when $iq \equiv \lambda$ is real, and our case of ID. For real λ in the second square bracket in (21) we will have sign plus between m^2 and $(m - k)^2$ - the same as in the first square bracket. For real λ we will have solution of the original equation (3) for the probability density (instead of wave function or epd) with the linear time-dependence of the dispersion. In our case of ID the sign inside the second square bracket is minus. This leads to elimination of the linear time-dependence and to the solution:

$$\Phi(t, k) = \exp\{-\frac{1}{2}(a^2 + w^2 t^2)k^2\} \quad (23)$$

which is the transform of (15). The elimination effect can be considered as interaction between m -tachyon and $(m - k)$ -antitachyon. This interaction we see after averaging. It will be very interesting to investigate in future - what kind of "battle of tachyons" takes place before averaging.

There is another way of looking at the random shooting of particle with the characteristic velocity $w = qa^{-1}$. Consider a pair of vortices with intensities q and $-q$, placed on the line perpendicular to real line in the complex plane: one above the real line, another below on the same distance $\sim a$. These vortices will move each other and the particle with velocity $\sim w$. This picture can be considered as coherent structure, produced by the flow of tachyons.

The 3-D generalization of ID is natural to do in terms of characteristic function with normalization condition $\Phi(t, 0) = 1$. For simplicity, we assume isotropy of initial probability. Substitution of 3-D Laplacian $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$ instead of $\partial^2/\partial x^2$ in equation (3) will not change the isotropy.

It means that in expressions (12) - (15) we have now $x^2 = x_1^2 + x_2^2 + x_3^2$ and prefactors are raised to the third power. In equation (23) we now have $k^2 = k_1^2 + k_2^2 + k_3^2$. Correspondingly: $\langle x_b \rangle = 0, (b = 1, 2, 3)$, $\langle x_b x_c \rangle = (a^2 + w^2 t^2) \delta_{bc}$, where δ_{bc} is the unit tensor. So, we get 3-D Gaussian distribution for the position of particle with indicated moments.

Tachyonic impulses can be at the core of the phenomena of quantum entanglement, as indicated before [3]. Consider system of n particles, positions of which $\mathbf{x} = (x_1, \dots, x_n)$ are initially correlated. We start again with 1-D case and 3-D generalization will follow. We assume that particles do not interact directly. The first possibility is that particles may not interact because of their nature. The second possibility is that we separate them initially, so that probability of direct collision is low (here 3-D generalization is handy). We choose initial condition such that mean positions $\langle \mathbf{x} \rangle$ will not change in time (see below). To simplify notation, we assume that $\langle \mathbf{x} \rangle$ is already extracted from \mathbf{x} . For the corresponding wave function $\Psi(t, \mathbf{x})$, instead of (3), we now have equation:

$$\frac{\partial \Psi}{\partial t} = i \sum q_j \frac{\partial^2 \Psi}{\partial x_j^2}, \quad q_j \equiv \frac{\hbar}{2m_j} \quad (24)$$

where summation is over $j = 1, \dots, n$ and m_j are masses of particles. Fourier transformation of (24) gives:

$$\partial \tilde{\Psi} / \partial t = -i [\sum q_j k_j^2] \tilde{\Psi} \quad (25)$$

Here $\tilde{\Psi}(t, \mathbf{k})$ is the transform of $\Psi(t, \mathbf{x})$ and $\mathbf{k} = (k_1, \dots, k_n)$ is the wave number vector. Solution of (25) is:

$$\tilde{\Psi}(t, \mathbf{k}) = \tilde{\Psi}_0(\mathbf{k}) \exp\{-it[\sum q_j k_j^2]\} \quad (26)$$

We assume initial Gaussian distribution for positions of particles [2]:

$$P_0(\mathbf{x}) = \frac{1}{(2\pi)^{n/2} (Det \mathbf{C})^{1/2}} \exp\{-\frac{1}{2} \mathbf{x} \mathbf{C}^{-1} \mathbf{x}\} \quad (27)$$

Here $\mathbf{C} = C_{jl} = \langle x_j x_l \rangle$ is the initial covariance matrix (symmetric) and \mathbf{C}^{-1} is the inverse matrix. In particular, $C_{11} = \langle x_1^2 \rangle = a_1^2$ is dispersion for position of the first particle with zero mean ($\langle x_1 \rangle = 0$), a_j corresponds to the particle j , $\rho_{jl}(0) = C_{jl}(a_j a_l)^{-1}$ (with $j \neq l$) is the initial correlation coefficient. Characteristic function for (27) is:

$$\Phi_0(\mathbf{k}) = \langle \exp\{i \mathbf{k} \mathbf{x}\} \rangle = \exp\{-\frac{1}{2} \mathbf{k} \mathbf{C} \mathbf{k}\} \quad (28)$$

Choosing initial wave function $\Psi_0(\mathbf{x}) = [P_0(\mathbf{x})]^{1/2}$, after simple calculation, we get transform:

$$\tilde{\Psi}_0(\mathbf{k}) = 2^{3n/4} \pi^{n/4} (Det \mathbf{C})^{1/4} \exp\{-\mathbf{k} \mathbf{C} \mathbf{k}\} \quad (29)$$

Formulas (26) and (29) give:

$$\tilde{\Psi}(t, \mathbf{k}) = 2^{3n/4} \pi^{n/4} (\text{Det} \mathbf{C})^{1/4} \exp\{-\mathbf{k}[\mathbf{C} + it\mathbf{q}\mathbf{I}]\mathbf{k}\} \quad (30)$$

Here $\mathbf{q}\mathbf{I}$ stands for diagonal matrix with elements q_j . This expression (apart from constant prefactor) is characteristic function for the joint epd of n complex-Gaussian trajectories like (10) with statistically independent velocities $v_j(t)$. It means, in particular, that $\langle v_1(t + \tau)v_2(t) \rangle = \langle v_1(t + \tau) \rangle \langle v_2(t) \rangle \equiv 0$ and mutual correlation function of these trajectories is equal to correlation of initial positions. Such simple is the quantum entanglement in the "complex world". In the real world situation is more complicated, as we will see below.

Consider characteristic function for $P(t, \mathbf{x})$ by using generalization of (18):

$$\Phi(t, \mathbf{k}) = \frac{1}{(2\pi)^n} \int d\mathbf{m} \tilde{\Psi}(t, \mathbf{m}) \tilde{\Psi}^*(t, \mathbf{m} - \mathbf{k}) \quad (31)$$

Substitution of (30) into (31) gives:

$$\Phi(t, \mathbf{k}) = \left(\frac{2}{\pi}\right)^{n/2} (\text{Det} \mathbf{C})^{1/2} \int d\mathbf{m} \exp\{-2\mathbf{m}\mathbf{C}\mathbf{m} + 2\mathbf{m}\mathbf{C}\mathbf{k} - 2it\mathbf{m}\boldsymbol{\varkappa} + it\mathbf{k}\boldsymbol{\varkappa} - \mathbf{k}\mathbf{C}\mathbf{k}\}$$

where we used symmetry of \mathbf{C} and introduced vector $\boldsymbol{\varkappa} = (q_1 k_1, \dots, q_n k_n)$. By substitution $\mathbf{m}\boldsymbol{\varkappa} = \mathbf{m}\mathbf{C}\mathbf{C}^{-1}\boldsymbol{\varkappa}$, the integral takes standard Gaussian form:

$$\Phi(t, \mathbf{k}) = \left(\frac{2}{\pi}\right)^{n/2} (\text{Det} \mathbf{C})^{1/2} \int d\boldsymbol{\mu} \exp\{-2\boldsymbol{\mu}\mathbf{C}\boldsymbol{\mu} + \frac{1}{2}\boldsymbol{\chi}\mathbf{C}\boldsymbol{\chi} + it\mathbf{k}\boldsymbol{\varkappa} - \mathbf{k}\mathbf{C}\mathbf{k}\}$$

where $\boldsymbol{\mu} = \mathbf{m} - \boldsymbol{\chi}/2$, $\boldsymbol{\chi} = \mathbf{k} - it\mathbf{C}^{-1}\boldsymbol{\varkappa}$. The linear time-dependence of dispersions is eliminated by the interaction of the tachyonic impulses, as can be anticipated. But, there are interesting details, which are hard to predict without calculations. The integration gives characteristic function, which corresponds to Gaussian distribution:

$$\Phi(t, \mathbf{k}) = \exp\left\{-\frac{1}{2}[\mathbf{k}\mathbf{C}\mathbf{k} + t^2\boldsymbol{\varkappa}\mathbf{C}^{-1}\boldsymbol{\varkappa}]\right\} \quad (32)$$

For $n = 1$, formula (32) reproduces (23). For $n = 2$ formula (32) gives:

$$\Phi(t, \mathbf{k}) = \exp\left\{-\frac{1}{2}[\sigma_1^2(t)k_1^2 + 2\rho(t)\sigma_1(t)\sigma_2(t)k_1k_2 + \sigma_2^2(t)k_2^2]\right\} \quad (33)$$

$$\sigma_1^2(t) = a_1^2 + w_1^2 t^2, \quad \sigma_2^2(t) = a_2^2 + w_2^2 t^2, \quad w_1 = q_1(a_1\beta_0)^{-1}, \quad w_2 = q_2(a_2\beta_0)^{-1} \quad (34)$$

$$\rho(t) = \rho_0 \frac{a_1 a_2 - w_1 w_2 t^2}{\sigma_1(t)\sigma_2(t)} \quad (35)$$

Here σ_1^2 and σ_2^2 are corresponding dispersions, ρ is correlation coefficient with initial value ρ_0 and $\beta_0 = (1 - \rho_0^2)^{1/2}$. The first important feature of

this distribution is the factor β_0 in (34) - increase of velocities due to initial correlation. In terms of vortices (see above), it could mean such interaction that, in average, gets them closer to real line.

Additionally, we get evolution of correlation coefficient $\rho(t)$ from initial value ρ_0 to asymptotic value $-\rho_0$ (when $t \gg \max\{a_1/w_1, a_2/w_2\}$). We can say that quantum vacuum "does not like" the measurement-imposed correlation and makes the inversion.

It will be interesting to play with these features experimentally. Particularly, it is important to investigate if the imposing of initial correlation can be made energy-efficient. Then the shooting effect with high velocities could be a potential mechanism for obtaining energy.

The 3-D generalization for the two-particle system is as follows. Positions of particles and wave numbers we denote by $x_{j,b}$ and $k_{j,b}$, where $j = (1, 2)$ indicate particle and $b = (1, 2, 3)$ corresponds to 3-D vector. Matrix of initial correlations now has two sets of indexes: $C_{jl,bc}$. In simplest case: $C_{jl,bc} = C_{jl}\delta_{bc}$. In this case we can use (33) with products k_1^2 , k_1k_2 and k_2^2 replaced by the scalar products of vectors, in particular, k_1k_2 is replaced by $k_{1,1}k_{2,1} + k_{1,2}k_{2,2} + k_{1,3}k_{2,3}$. From such generalization of (33) it follows: $\langle x_{j,b} \rangle = 0$, $\langle x_{1,b}x_{1,c} \rangle = \sigma_1^2\delta_{bc}$, $\langle x_{2,b}x_{2,c} \rangle = \sigma_2^2\delta_{bc}$, $\langle x_{1,b}x_{2,c} \rangle = \rho\sigma_1\sigma_2\delta_{bc}$.

In general case formula (32) gives:

$$\langle x_{j,b}x_{l,c} \rangle = C_{jl,bc} + t^2 q_j q_l C_{jl,bc}^{-1} \quad (36)$$

In particular:

$$\langle x_{1,1}^2 \rangle = C_{11,11} + t^2 q_1^2 C_{11,11}^{-1} \quad (37)$$

Velocity of the shooting in (37) is determined by the term of the inverse covariance matrix, which is multiplied by $x_{1,1}^2$ in the initial probability density (27). For the two-particle system this term, according to (34), is proportional to $(1 - \rho_0^2)^{-1}$. For $n \geq 3$ velocity of the shooting depends of all initial correlation between particles.

The presented results can be also obtained and interpreted in terms of the Wigner function (see, for example, recent book [4] and references therein):

$$W(t, x, p) = \frac{1}{2\pi\hbar} \int dr \Psi(t, x - r/2) \Psi^*(t, x + r/2) \exp\left\{\frac{ipr}{\hbar}\right\} \quad (38)$$

Here we consider the 1-D case for one particle and p is additional (momentum) variable. The probability density is given by integral:

$$\Psi\Psi^* = \int dp W \quad (39)$$

Solution (5) [or (6)] corresponds to:

$$W(t, x, p) = W_0\left(x - \frac{pt}{m}, p\right) \quad (40)$$

Initial condition (12) gives:

$$W_0(x, p) = \frac{1}{\pi\hbar} \exp\left\{-\frac{x^2}{2a^2} - \frac{2a^2 p^2}{\hbar^2}\right\} \quad (41)$$

Using (40), we have:

$$W(t, x, p) = \frac{1}{\pi\hbar} \exp\left\{-\frac{(x - ptm^{-1})^2}{2a^2} - \frac{2a^2 p^2}{\hbar^2}\right\} \quad (42)$$

Integration (39) of (42) reproduces (15). In addition, we get interpretation of the random shooting (16) in terms of variable p . Let us note that formula (15) for one particle was known before (see references in [4]), but interpretation was different. Expression $(a^2 + w^2 t^2)^{1/2}$ (in our notation) was interpreted as trajectory [4]. In our interpretation, the motion is with constant (random) velocity (16).

The n -particle system has analogous description by the Wigner function with interpretation of general formulas (32) and (36) in terms of corresponding p -variables. It is helpful to have in mind both approaches (Schrödinger and Wigner) in order to develop intuition, which unites tachyonic impulses and p -variables.

We plan detailed investigation of systems with three or more particles by using presented above general results.

Further studies of ID can also open new perspective in quantum field theory. I thank V. I. Tatarskii for useful discussion.

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